

Enriching Math Using Chess

Background

I started to teach chess to my son when he was five years old and soon noticed that the relationship between mathematics and chess is one of those generally presumed truisms. I was not able to find a book of junior level math and chess hybrid problems for my son to work on however. In 1995, after seriously looking into the possibility of writing one myself, I pioneered the idea of integrating chess symbols and their values into math. For the first time, elementary students were able to visualize how concepts in both math and chess are related. The idea of using chess symbols and their point values and chess moves directly in puzzles characterizes what I believe is a breakthrough that permits a true integration of math and chess, since the puzzles are no longer limited to only traditional chess puzzles.

Later, I created worksheets with a 2-column system; chess on one side and math puzzles on the other. Students are given the opportunity to see the relationship between math and chess side by side. I also used chess symbols directly in arithmetic operations. The effect is that simple, one-step questions instantly become abstract and symbolic multi-step questions that require children to analyse the problem and take necessary steps to understand the abstract concept before coming up with a solution – excellent training for critical thinking and problem solving.

I have been fortunate to have the opportunities to personally field-test these problems at my own learning centre since I have been teaching math from kindergarten to grade 12 over the past 10 years. This unique experience has allowed me to obtain a wide spectrum of feedback from different student backgrounds. The article will attempt to show how different kinds of mathematical chess puzzles can be produced and their potential benefits for learning outcomes.

Chess benefits children

Why chess fascinates children? Dr. Montessori observed that younger children were intensely attracted to sensory development apparatus. Chess being hands-on and multi-sensory, involves coordination between eyes, brain and hands in multi-direction, and embodies concepts that are non-linear when compared to most video and computer games. As young as five, my son was thrilled to be able to capture my chess pieces as his reward that perhaps was the sensory experience kept him hungry for more games.

In addition to have fun, playing the game of chess has been shown to improve in cognitive and critical thinking skills, reasoning and problem-solving abilities, focusing, visualizing, analytical and planning skills. These conclusions have been backed up by educational research papers (1).

How are my puzzles different from others?

In the past, many chess puzzles have been published and as for strictly chess problems the standard has long been set by Sam Loyd, the "Puzzle King", composer of some of the most paradoxical, almost phenomenal, chess problems. The most recent mathematics and chess related book entitled Mathematics and Chess has 110 entertaining problems and solutions. (2) Almost all of these published puzzles, old and new, are related to the moves of chess pieces and the majority of them are considered to be too difficult for most of elementary students to comprehend.

A math textbook series called "Challenging Mathematics" (3) has chess as part of a logic section but the chess content itself is stand alone and is not integrated with any math concepts or math problems.

I used chess symbols, chess values, chess moves, chessboard, algebraic notation, chess set-up, attacking and defending counting, order of exchanges, and chess rule etc. to create mathematical chess puzzles. The fundamental difference between my mathematical chess puzzles and those traditionally published is that chess symbols, moves, and values are integrated into math to create problems in patterns, logic, geometry, counting paths, relation, arrangement, numeration, and even data management. In this way children from pre-kindergarten through elementary school, while learning chess, are provided an opportunity to explore mathematical puzzles by making use of the very basics of chess knowledge.

The puzzles are designed to enhance math ability using chess as a teaching tool. It is not intended to substitute for instruction in school math but rather to serve as enrichment or supplemental material. Children learn best by playing games, Math + Chess = a fun way of learning math.

How chess and math are integrated

The creation of math and chess problems require one to have through understanding of chess knowledge and also the school mathematics curriculum at each grade level, Only one processes these qualifications and creative mind, can a meaningful mathematical chess puzzle be created. Integrated chess and math problems are created using the following principles:

(1) Chessboard and chess pieces.

The chessboard is symmetric in main diagonals in terms of its colour. The chessboard is made of four identical small boards if it is divided by one horizontal line and one vertical line going through the centre. The set-up position of chess pieces is symmetric between Black and White. The chess pieces set-up position on either side is palindrome excluding the king and queen.

The ranks and files are related to coordinates. When a piece is being attacked or defended, it requires some arithmetic calculations in terms of the number of attacking and defending pieces and this is the first lesson a child would learn in counting numbers.

(2) Chess moves

Rook's move is a slide motion (left/right, up/down) in geometry. The between moves of rook before reaching the destination is similar to the concept of commutative property. For example, to reach from a1 to h1(7) = a1 to c1 (2) + d1 to h1 (5) = h1 to d1 (5) + c1 to a1 (2).

To figure the "best" move, one needs to find out all possible paths, the similar concept in math would be to use the factor tree to find out the prime factors of a number, for example to find out the prime product of a number 32. One could use a factor tree to do it.

How to checkmate an opponent? If rook is at $a1$ and is free to make moves along file a and rank 1, what has to be considered before moving? To see if there any opponent's pieces intersecting with rook would be like to find out what would be

y when $x = 1$? The checkmate positions are actually the intersections of ranks or files, which are very similar to the concept to solve equations by using graphic method.

To find the common squares (squares both pieces could control) is similar to the idea of finding common factors of two numbers.

One would think that chess perhaps has nothing to do with fractions since all moves are all in whole numbers? Why queen is the most powerful piece and usually we move pieces toward the centre? They all have something to do with the ratio $a/64$ where a is the number of squares under control.

When chess players check possible moves, the view encompasses a circular locus through 360° . For example, when checking a rook's possible moves, a player scans the following angles: $0^\circ, 90^\circ, 180^\circ, 270^\circ$. In other words, the rook's move is equivalent to rotate rook in four directions. The same rotation concept is true for bishop, queen, and king.

(3) Chess Symbols and Chess Values

The Use of Chess Symbols as Constants

English letters such as x, y, z, \dots are normally used to represent unknown numeric values. These unknown letters are also called variables and they normally do not have singly defined values. On the other hand, each chess symbol has definitely defined meaningful point value that is related to each piece's strength in the chess game. The point system is the static value of a piece and generally serves as a guide to making trades with the opponent. Take a look at the following example.

Let $x = 1, y = 3$ then $x + y = 1 + 3 = 4$

In the above particular example, x is 1 and y is 3 but x does not always have to be 1 nor does y always have to be 2.

If we use chess symbols in the above example, we get

$$\text{♟} + \text{♗} = 4$$

The above pawn and bishop have specifically defined values 1 and 3 respectively and will not change their values just because the problem is different, in other words, they are constants.

By contrast, in algebra, students would substitute x or y with different values when given values are changed. In other words, in algebraic equations, the values of x and y change in terms of the particular problem. To compare the substitution values for chess symbols and algebraic variables, we realize that there is a difference in the chess symbols substitution - it is intuitive for children since the value of each chess symbol is pre-defined and hence has an implicit meaning to them.

The idea of using chess symbols is to teach children how to transfer from a concrete object into abstract concept. For example, a concrete object (like a bishop chess piece) could be associated with a symbol ♖, which in turn could be substituted with a value of 3. This process of learning and thinking skill learned is in line with the concept of Montessori teaching.

To use animal figures or any other symbols such as x, y, z in mathematical chess puzzles would be less meaningful to children when compared to the use of chess symbols in math and chess integrated puzzles problems. Children do not get confused on the use of chess symbols, not will they become handicapped when they learn the variables in high schools just because they learn substitution at young age. The chess symbols are only used as “pictograms”, i.e. as representations of values. They understand the spaces in an arithmetic expression where these chess symbols occupy should have been numbers in the first place when used in arithmetic operations.

The other reason of using chess symbols in mathematical chess puzzles is the chess symbols themselves representing movements and coincidentally some of the directions of movements resemble some arithmetic operators, for example rook can move up and down and left right and thus its trace of capable moves looks like a + sign.

Each chess symbol has a specially defined direction of move and these directions of moves are “imbedded” with each piece. I have taken the advantage of chess pieces moves and defined them as follows:

Addition/Subtraction = Rook (Could also be queen or king)

Multiplication = Bishop (Could be also be queen or king)

Division = King (Opposition of 2 kings)

Chess Values Used

The values of chess symbols are the same as the ones used in the Teaching Manual published by the Canada Federation of Chess (4).

The cancellation technique to count the points of chess pieces would be very similar to the concept of subtraction property of an equation.

Chess symbols and values are integrated in arithmetic operations to create a new type of problems. My purpose of using chess symbols is to create more interesting questions and encourage children to think ahead and use spatial-temporal reasoning to solve problems.

Each chess piece has been assigned a different point (value), for example, the following are values assigned to chess symbols.

 (king) = 0 point	 (knight) = 3 points	 (rook) = 5 points
 (pawn) = 1 point	 (bishop) = 3 points	 (queen) = 9 points

My experience of using chess values in teaching arithmetic operations is very positive. Elementary students who have not learned variables but have worked on my worksheets using chess symbols have absorbed the concept of algebraic variables or substitution in a natural and intuitive way. There is no need to explain the concept of variable other than mention the values of chess pieces, for example,

$$\text{♖} + 5 = \underline{\hspace{2cm}}$$

Examples

Listed below are a few examples I created to show the relations between math and chess.

Example 1 Addition and Subtraction

$\begin{array}{r} \square \\ \hline + \text{♞} \\ \text{♔} \end{array}$	$\begin{array}{r} \square \\ \hline - \text{♖} \\ \text{♔} \end{array}$	$\begin{array}{r} \square \\ \hline + \text{♖} \\ 6 \end{array}$	$\begin{array}{r} \square \\ \hline - \text{♖} \\ 6 \end{array}$
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The above problem is the one I designed to go against conventionally designed worksheets, which is always from left to right in linear fashion. One could work out the above problem from the bottom to top and then from top to down in multi-direction.

Example 2 Multiplication

$$\begin{array}{ccc}
 \frac{25}{\square \times} = 5 & \begin{array}{c} \square \\ \times \\ \square \\ \hline \end{array} & \frac{25}{\square \times} = 5 \\
 & \begin{array}{c} \swarrow \quad \parallel \quad \searrow \\ \square \\ \hline \end{array} & \\
 \square \times \square = \square = \square \times \square & & \\
 \begin{array}{r} \times \square \\ 5 \overline{) 25} \end{array} & \begin{array}{c} \swarrow \quad \parallel \quad \searrow \\ \square \\ \times \\ \square \end{array} & \begin{array}{r} \times \square \\ 5 \overline{) 25} \end{array} \\
 \parallel & & \parallel \\
 \begin{array}{r} \square \overline{) 25} \\ \times \square \end{array} & & \begin{array}{r} \square \overline{) 25} \\ \times \square \end{array}
 \end{array}$$

The above problem is created with the mind that children do not really learn math in a sequential way of addition, subtraction, multiplication, or division in real life. So here you could see how I incorporate the idea of multiplication different formats of computing. The purpose of this worksheet is to learn multiplication but expose children in on how multiplication could be written in different ways. A simple multiplication problem is changed to a two-step problem in multi-direction, multi-operation, and multi-concept learning.

Example 3 Logic

Chess Symbol	Logic Training														
<p>A new Chess Symbol is defined as follows:</p> <table border="1" data-bbox="162 420 779 829"> <thead> <tr> <th>Chess figurines</th> <th>Chess symbols</th> </tr> </thead> <tbody> <tr> <td></td> <td>\div (Opposition)</td> </tr> <tr> <td></td> <td>\dagger</td> </tr> <tr> <td></td> <td>\perp</td> </tr> <tr> <td></td> <td>\times</td> </tr> <tr> <td></td> <td>$*$</td> </tr> <tr> <td></td> <td>∇</td> </tr> </tbody> </table>	Chess figurines	Chess symbols		\div (Opposition)		\dagger		\perp		\times		$*$		∇	<p>In the following equation, observe the chess symbols on the left and fill in each \bigcirc with a number.</p> <p>If $\dagger + \dagger = 10$ then $\div + \dagger = \bigcirc$</p> $ \begin{array}{r} \dagger \quad * \\ + \quad \dagger \quad \div \\ \hline \bigcirc \quad \div \quad * \end{array} $
Chess figurines	Chess symbols														
	\div (Opposition)														
	\dagger														
	\perp														
	\times														
	$*$														
	∇														

<p>Use the above Chess Symbol table, find the following pattern:</p> <p>Z, \div, O, ∇, T, \perp, T, \times, F, \dagger, _____, $*$</p>
<p>Use the above Chess Symbol table, find the following pattern:</p> <p>0, \div, 1, ∇, __, \perp, 3, \times, 5, \dagger, __, $*$</p>

The above problem demonstrates that not only children will not get confused on traditional chess symbols used in arithmetic expression in my workbook, they could be led to use additional “creative” chess symbols and are able to solve them correctly. This problem is suitable to grade 3 and above students.

Most students could not solve the last 2 puzzles but they appreciate the sophistication of the problems when I explained to them the logic relationships. The last problem is the one using a combination of chess symbols and their values.

The use of chess values are much like the use of monetary values. When chess or money figures are seen by children, they both represent some pre-defined meaningful values. The following is an example where the values of chess pieces could be monetary values and "Total Points" could be the sum of total money.

Example 4 Table Values

Fill in the different number of chess pieces to come up with each total.

Number of 	Number of 	Number of 	Total points
1	1	1	9
3	2	0	9
0	3	0	9
<input type="text"/>	<input type="text"/>	<input type="text"/>	10
<input type="text"/>	<input type="text"/>	<input type="text"/>	10
<input type="text"/>	<input type="text"/>	<input type="text"/>	11
<input type="text"/>	<input type="text"/>	<input type="text"/>	12
<input type="text"/>	<input type="text"/>	<input type="text"/>	13
<input type="text"/>	<input type="text"/>	<input type="text"/>	14
<input type="text"/>	<input type="text"/>	<input type="text"/>	15

Example 5 Equation

The following examples demonstrate how chess symbols and chess values are integrated with arithmetic operations to facilitate the critical thinking skill.

$\text{♔} + \text{♞} + x = 54$ $x = \underline{\hspace{2cm}}$
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Example 6 Addition and Subtraction, If Then - Else

$$\begin{array}{r}
 10 - \text{king} = \square \\
 + 5 + \text{king} = \square + \\
 \hline
 \square \qquad \qquad \square
 \end{array}$$

If $10 + \text{king} = \square$, then $9 + \text{king}$ must be \square .

If $\text{king} + 10 = \square$, then $\text{king} + 9$ must be \square .

Example 7 Cross Multiplication

$$\begin{array}{ccc}
 \text{king} & & 8 \\
 || & & || \\
 \square & & \square \\
 \times & \times & \times \\
 \square & & \square \\
 \square & + & \square = 6
 \end{array}$$

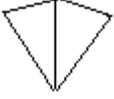
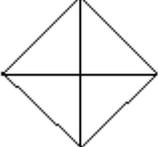
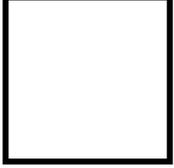
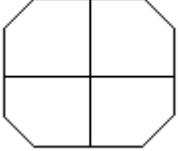
Example 8 Multiplication and division

$\begin{array}{r} \times \text{♔} \\ \times 2 \\ \hline 18 \div 2 = \square \end{array}$	$\begin{array}{r} \times \text{♖} \\ \times \text{♖} \\ \hline \square \div 5 = \square \end{array}$
$\begin{array}{r} \times \text{♔} \\ \times \text{♖} \\ \hline \square \div 9 = \square \end{array}$	$\begin{array}{r} \times \text{♔} \\ \times \text{♔} \\ \hline \square \div \text{♔} = \square \end{array}$
$\begin{array}{r} \times \text{♔} \\ \times \text{♘} \\ \hline \square \div \text{♘} = \square \end{array}$	$\begin{array}{r} \times \text{♔} \\ \times \text{♘} \\ \hline \square \div \text{♔} = \square \end{array}$
$\begin{array}{r} \times \text{♖} \\ \times \text{♘} \\ \hline \square \div \text{♘} = \square \end{array}$	$\begin{array}{r} \times \text{♖} \\ \times 8 \\ \hline \square \div \text{♖} = \square \end{array}$

Will the above operations cause confusion to children, the answer is no. $\text{♔} \times \text{♘}$ will not make sense if it is explained literally as a queen times a knight. However if it is translated into numerals 9 times 3 then the product must be 27 which is very logic and children understand that they are working on 9×5 not the product of $\text{♔} \times \text{♘}$.

A similar type of logic question is as follows: If $2 \# 3$ is defined as $2 + 2 \times 3$ then what is $3 \# 4$? Normally $2 \# 3$ will not make any sense since it is not a valid arithmetic operator but if we define it clearly then it becomes workable.

Example 9 The following is a puzzle that requires the knowledge of chess moves.

Filling in  by a chess piece	Geometric shapes
	
	
	
	

Example 10 Use chess moves to solve the following puzzle.

	?		14	
?				21
				
?				28
	42		?	

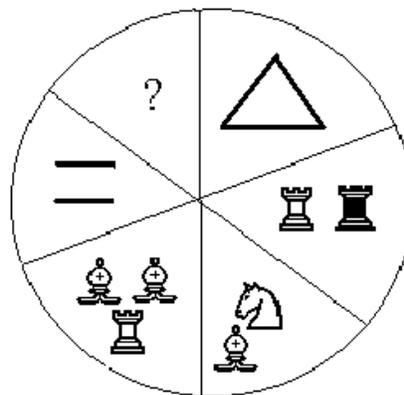
On the first look, lots of students are not able to solve it, why? Students are so used to do computation from left to right and this question has to be solved in an unconventional direction.

Example 11 Use chess symbol moves to solve the following puzzle.

If $2 \text{ ♖ } 3 = 5$ then $2 \text{ ♜ } 3$ is = _____

Surprisingly, some of my students have no trouble to solve the above puzzle.

Example 12 Use chess symbol moves to solve the following puzzle.



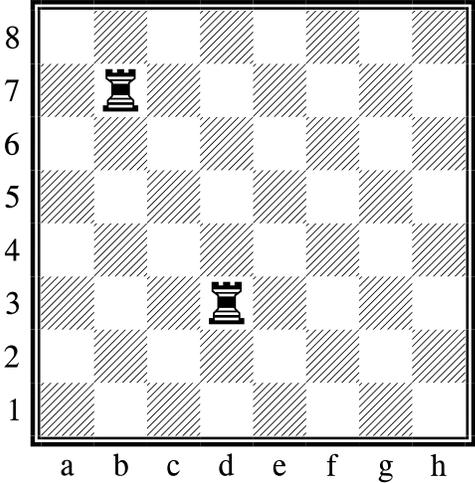
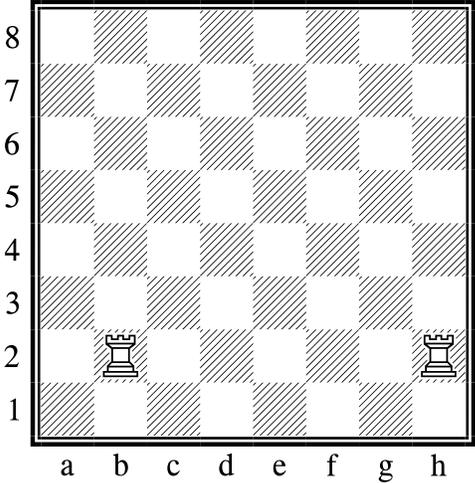
Example 13

If $\text{♔} \div \text{♝} = \text{♖}$
Then what is $\text{♔} \div \text{♖} = ?$

This problem can not be solved using chess values; students tried it and knew it. So what is the trick behind the idea of this puzzle?

Example 14

Rook Path

<p>Cross mark (X) the square(s) where all rooks could share the common squares.</p>  <p>A chessboard with columns labeled a-h and rows labeled 1-8. A rook is on square b7 and another rook is on square d3.</p>	<p>Find the common factors of the following numbers.</p> <p>12, 24</p> <p>13, 26</p>
<p>Cross mark (X) the square(s) where all rooks could share the common squares.</p>  <p>A chessboard with columns labeled a-h and rows labeled 1-8. A rook is on square b2 and another rook is on square h2.</p>	<p>Find the common factors of the following numbers.</p> <p>11, 121</p> <p>3, 26</p>

Chess moves problems could be applied to math concept, I give the above example to demonstrate the idea with chess problem on one side and the other side is the math problem.

Summary

I have found the idea of using chess symbols very helpful for elementary students – they could learn to solve mathematical chess puzzles using their chess knowledge.

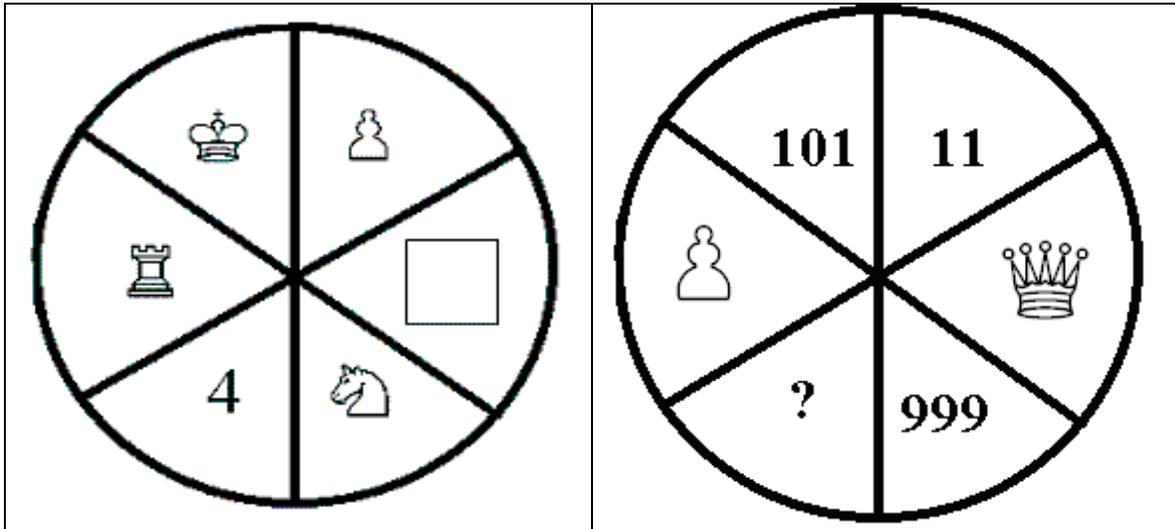
Many children could not play chess well when against someone but feel very proud that they could solve mathematical chess puzzles. Mathematical chess puzzles provide some children additional opportunities that they could challenge themselves. For this reason, I give prizes to chess winners and also to puzzles solvers.

The most interesting in using chess symbols is that the chess symbols themselves not only possess pre-defined values but also have the implied meaning of movements and these two special characters allow me to create some very interesting mathematical puzzles with pizzazz.

By using chess symbols, a simple one-step arithmetic problem could become a multi-step problem, as this result, chess symbols and values offer children more opportunities to work on another type of questions which could simulate children's brain cell and improve their problem-solving ability. So the benefits of working on these types of problems is double edged- improving chess knowledge and also mathematical problem-solving ability.

The mathematical chess puzzles created by me are not just mechanically substituted numbers with chess symbols. Many mathematical chess puzzles created also involve pattern, sequence, geometry, set theory, and logic etc. In other words, the integration is very diversified and also involves multi-direction visualization. I would give the following examples to demonstrate on how chess symbols and values are presented in the following pattern-like puzzles with the approach of multi-direction and multi-sensory to conclude this article.

Example 15 Find values to replace? or fill in \square .



Reference

- (1) Chess in Education Research Summary, Dr. Robert Ferguson. Details see below http://www.quadcitychess.com/benefits_of_chess.html
- (2) Mathematics and Chess, by Miodrag Petkovic, Dover Publications, Inc. Mineola, New York, 1997
- (3) Challenging Mathematics, Cheneliere/McGraw-Hill, Michael Lyons, Robert Lyons, 1995, Montreal, Quebec, Canada
- (4) Chess Teaching Manual, IM Tom O'Donnell, the Chess Federation of Canada.